Acoustic source inversion to estimate volume flux from volcanic explosions

Keehoon Kim1, 2, David Fee1, Akihiko Yokoo3, and Jonathan M. Lees4

1Wilson Infrasound Observatory, Alaska Volcano Observatory, Geophysical Institute, University of Alaska Fairbanks, Fairbanks, Alaska, USA, 2Now at Geophysical Monitoring Program, Lawrence Livermore National Laboratory, Livermore, California, USA, 3Institute for Geothermal Sciences, Kyoto University, Kumamoto, Japan, 4Department of Geological Sciences, University of North Carolina at Chapel Hill, Chapel Hill, North Carolina, USA

Abstract We present an acoustic waveform inversion technique for infrasound data to estimate volume fluxes from volcanic eruptions. Previous inversion techniques have been limited by the use of a 1-D Green's function in a free space or half space, which depends only on the source-receiver distance and neglects volcanic topography. Our method exploits full 3-D Green's functions computed by a numerical method that takes into account realistic topographic scattering. We apply this method to vulcanian eruptions at Sakurajima Volcano, Japan. Our inversion results produce excellent waveform fits to field observations and demonstrate that full 3-D Green's functions are necessary for accurate volume flux inversion. Conventional inversions without consideration of topographic propagation effects may lead to large errors in the source parameter estimate. The presented inversion technique will substantially improve the accuracy of eruption source parameter estimation (cf. mass eruption rate) during volcanic eruptions and provide critical constraints for volcanic eruption dynamics and ash dispersal forecasting for aviation safety. Application of this approach to chemical and nuclear explosions will also provide valuable source information (e.g., the amount of energy released) previously unavailable.

1. Introduction

Volcanic eruptions produce pressure disturbances that propagate as seismic waves into the Earth and low-frequency acoustic waves (termed infrasound) into the atmosphere. As inversion of seismic signals has provided information on subsurface mass transfer associated with volcanic unrest [Ohminato et al., 1998; Chouet, 2003; Haney et al., 2012; Chouet and Matoza, 2013], waveform inversion of infrasound data can provide critical constraints on eruption processes in the upper conduit and vent [Fee and Matoza, 2013].

Although it is well known that crater morphology and local volcanic topography can distort the infrasound wavefield significantly [Kim and Lees, 2011; Lacanna and Ripepe, 2013; Kim and Lees, 2014], the topographic propagation effects so far have not been considered in source inversion. Acoustic waveform inversion for volcanic infrasound has conventionally been performed assuming a free space or half space; topography is neglected for sound propagation and the simplest 1-D Green's function (depending only on the source-receiver distance) is considered [Johnson et al., 2008; Kim et al., 2012; Johnson and Miller, 2014]. Recently, topographic scattering was partly considered in acoustic source inversion for strombolian eruptions at Stromboli Volcano [Iemma et al., 2015], but an axisymmetric assumption was made on the topography, which is unrealistic for most volcanoes. Applicability of a 3-D Green's function for volcanic infrasound inversion has thus far not been fully explored nor validated by field data.

Here we introduce a waveform inversion technique for an acoustic source (volume flux in this case) where the acoustic Green's functions are computed via a 3-D finite difference time domain (FDTD) method that accounts for realistic topographic scattering. We apply this inversion technique to explosive volcanic eruptions at Sakurajima Volcano, Japan. Our numerical simulations using a high-resolution digital elevation model (DEM) produce excellent waveform fits to field observations and show that reflection and diffraction of sound from the crater rim are significant. The inversion results demonstrate that acoustic source inversion without consideration of topographic effects leads to incorrect estimates of volume flux histories from explosions.
Figure 1. Three-dimensional digital elevation map of Sakurajima Volcano. Five infrasound sensors (inverted triangles) were deployed around the edifice with good azimuthal coverage during 17–24 July 2013. Explosions originated from Showa Crater during the deployment period.

2. Data Acquisition

Sakurajima Volcano is a postcaldera volcano formed near the southern edge of Aira Caldera in southern Kyushu, Japan. Showa Crater (Figure 1), on the southeast side of the summit crater (Minamidake), has been the focus of volcanic explosions since 2006. Eruptive activity at Showa Crater is characterized by archetypical vulcanian eruptions producing small (<3 km above sea level) ash-laden plumes and large volcanic bombs (meter size) ejected to distances of 1–2 km [Yokoo et al., 2013] (Figure 1). A temporary infrasound network consisting of five stand-alone digital infrasound sensors was deployed at 2–6 km range by the University of Alaska Fairbanks between 17 and 24 July 2013 [Fee et al., 2014]. During the field campaign, Sakurajima produced vulcanian-style explosions from Showa crater that were typical of ongoing activity at the volcano since 2006 [Iguchi et al., 2013]. Infrasound signals coincident with these explosions are generally characterized by a high-amplitude, short-duration onset followed by lower amplitude, long-duration coda [Fee et al., 2014]. The impulsive onset with a strong compression/rarefaction indicates the destruction of a magma plug and sudden release of pressurized gas pockets [Yokoo et al., 2013]. Infrasound signals during the few seconds after the onset consist of complex waveforms (Figure 2) indicating sustained fragmentation processes and/or complicated wave propagation attributed to the crater morphology. These complex waveforms often transition into continuous lower intensity tremor or jetting, suggesting persistent open-vent activity [Fee et al., 2014; Matoza et al., 2014].

3. Numerical Modeling

It is well documented that crater morphology and local intervening topography can significantly affect infrasound propagation [Matoza et al., 2009; Kim and Lees, 2011, 2014; Lacanna et al., 2014]. To examine waveform distortion by local topography, we perform numerical modeling of acoustic wave propagation using a 3-D FDTD method [Ostashev et al., 2005]. A high-resolution (1 m) DEM acquired by airborne laser surveys in October 2013 (provided by the Japanese Ministry of Land, Infrastructure, Transport and Tourism) is used to represent the air-solid interface where rigid boundary conditions are applied. We assume a homogeneous and nonmoving atmosphere with a constant speed of sound (349 m/s), since atmospheric heterogeneity should have limited influence in this local range (<7 km) [Johnson and Ripepe, 2011; Kim and Lees, 2014]. Intrinsic attenuation and nonlinear phenomena are also ignored. The spatial grid interval is 8 m, corresponding to 30 grid nodes per shortest wavelength to ensure numerical accuracy [Wang, 1996]. Full 3-D numerical modeling has thus far not been used extensively for infrasound studies because of the computational burden.
In order to increase computational performance, we adapted a parallel algorithm exploiting many-core GPUs [Micikevicius, 2009]. The GPU-accelerated FDTD algorithm significantly reduces computation time (∼1 h for the computational mesh of 500 million cells) and makes 3-D numerical modeling of infrasound practical even on a personal computer [Kim and Lees, 2014].

Characteristic infrasound waveforms and sound propagation effects are illustrated in Figure 2. A typical explosion signal (occurring at 22:33:14 UTC on 20 July 2013) is selected for comparison, denoted by red lines in Figure 2a. The numerical model prediction across the network is denoted by gray lines (Figure 2a). The synthetic infrasound signals are excited by a Gaussian-like pulse, termed Blackman-Harris window function [Harris, 1978] with a cutoff frequency of 1 Hz, inserted at the center of Showa Crater (Figure 2b) and denoted by the location SRC (Figure 2c). While the pressure-time history at SRC does not have negative pressure, the synthetic waveforms predicted across the network (ARI to SVO) show a significant rarefaction and subsequent oscillation that likely arises from diffraction and reflection during propagation (Figure 2a). The synthetic waveforms show excellent agreement with infrasound records from the explosion event, strongly suggesting that the observed rarefaction and oscillation are attributable to path effects rather than source complexity. Figure 2c compares synthetic waveforms at KUR to the synthetics predicted at the location TMP (Figure 2b), which is on the same vertical profile as KUR (Figure 2c) but is adjacent to Showa Crater. TMP waveforms show significant negative pressure after the compressional onset indicating that the rarefaction waves develop from sound diffraction at the crater rim [Kim and Lees, 2011]. The oscillatory feature in the waveforms are further intensified at station KUR reflecting path effects over the intervening topography. These results demonstrate waveform distortion by crater geometry and topography, which are significant and must be taken into consideration in source inversions.
4. Inversion Method

Waveform inversion techniques are common in seismology, with 3-D Green's functions computed to help determine the seismic moment tensor and force vector [Chouet, 2003; Kim et al., 2014]. Here we propose an analogous method for acoustic waves where the moment tensor and seismic Green's functions are replaced with the monopole source term and acoustic Green's functions, respectively. An acoustic monopole is a point approximation of volume expansion/contraction in the atmosphere [Wouff and McGetchin, 1976]. In conventional methods [Johnson et al., 2008; Johnson and Miller, 2014; Kim et al., 2012], the monopole strength is typically inverted assuming a homogeneous half-space atmosphere where acoustic pressure depends only on distance from the source. This assumption is questionable since many volcanoes worldwide have pronounced crater morphology and nearby topography; thereby, ignoring topographic scattering of acoustic waves may lead to large errors in the inversion of the source time history. Our inversion technique accounts for these propagation effects by computing 3-D numerical Green's functions using the FDTD method and high-resolution DEM. We hereafter refer to the conventional method as the 1-D method and our method as the 3-D method in accordance with the geometric dimension of the Green’s function used in the inversions.

Acoustic wavefields excited by a monopole can be expressed by the convolutional equation according to the representation theorem [Morse and Ingard, 1986]:

\[ p(x, t) = G_0(x, t; x_0, t_0) \ast Q(t). \]  

(1)

where \( p \) is the acoustic pressure measured at position \( x \) and time \( t \), \( G_0 \) is the Green’s function excited by a monopole source at location \( x_0 \) and time \( t_0 \), the symbol \( \ast \) denotes convolution operator, and \( Q \) is monopole strength corresponding to volume flux history (kg/s). Equation (1) can be rewritten in matrix form [Ohminato et al., 1998] consisting of the data vector \( d \) of recorded pressure at each station, system matrix \( G \) containing the Green’s functions, and column vector \( m \) of the monopole source:

\[ d = Gm. \]  

(2)
Figure 4. Inverted volume flux histories. (a) Volume flux history obtained for the best fit source location. The flux curve inverted by our 3-D method (red line) is compared to that computed by the conventional 1-D method (gray line). Peak volume flux in the 1-D method is 1.76 Mm³/s, which is overestimated up to 155% compared to the 3-D method (1.13 Mm³/s). (b) Cumulative volume flux corresponding to the volume flux curves in Figure 4a. The 1-D method underestimates the total volume flux (1.57 Mm³) up to 51% compared to our 3-D method (3.04 Mm³).

Equation (2) can be solved for \( m \) by minimizing the squared error [Ohminato et al., 1998; Kim et al., 2014]. The waveform misfit (residual) \( R \) is defined as

\[
R = \frac{d - Gm}{d^T d}.
\]

5. Results

Volume flux inversion is performed for the same event shown in Figure 2a, and the optimal source location is determined using a grid search method. We compute the Green's functions for each grid node to receiver location using a Blackman-Harris function (with a corner frequency of 2 Hz) as the source time function and subsequently obtain the waveform misfit for the inversion at each respective grid node. The grid node with the minimum waveform misfit represents the most likely source location. We use a grid of 600 m × 600 m × 240 m in the east, north, and the vertical directions, respectively, around Showa Crater with 40 m node spacing. Figure 3a shows the results of the grid search. The minimum residual solution (indicated by the white star) lies at the bottom of Showa Crater, consistent with previous work [Yokoo et al., 2013; Kim and Lees, 2014]. The residual distribution may provide a measure of the uncertainty in source location. Considering small residual errors \( R < 0.12 \) in the cross-section views (Figure 3a), the vertical uncertainty is larger than the horizontal uncertainty due to the limited vertical coverage of the network: infrasound sensors are deployed only on the ground surface. Waveform fit at the optimal source location is shown in Figure 3b. Observed waveforms across the network are fit remarkably well by synthetic waves, with a misfit of 0.05 (5% of the squared sum of observed signals). This small waveform misfit and reasonable source location justify the monopole source model and reliability of the inversion solution.

We now compare the results from our 3-D inversion method to those of the conventional 1-D method. In the 1-D method, the propagation medium is assumed to be a homogeneous half space where the receivers are in the line of sight to the source [Moran et al., 2008; Johnson et al., 2008; Kim et al., 2012; Ripepe et al., 2013]. Volume flux history is calculated solely through integration of the infrasound data from station KUR [Johnson and Miller, 2014]. KUR is selected since it is in the most unobstructed line-of-sight path from Showa Crater. Each line-of-sight path from the crater to the other stations is likely blocked by the volcano summit or irregular terrain [Fee et al., 2014].

Figure 4 shows the volume flux history and cumulative volume flux resulting from our 3-D inversion at the best fit source location. The volume flux shows a rapid increase at the onset of the explosion, and then gradual decrease over the next 5–10 s, which can be expected from the results of the high-speed imaging [Chouet et al., 1974] of and the multiphase flow modeling [Clarke, 2013] for explosive eruptions. However, the solution from the 1-D method exhibits significant oscillations in volume flux (Figure 4a) that are a consequence of assuming the 1-D Green's function. Further, peak volume flux using the 1-D Green's function is overestimated up to 155% compared to the 3-D method (Figure 4a), while the total erupted volume (Figure 4b)
is underestimated up to 51%. Other explosions at Sakurajima show similar inversion results and volume flux histories.

In this study, we do not validate the inferred volume flux history against other independent observations, since a direct measurement of volume flux is not available. Instead, and similar to the seismic case, the validity of the inversion results are statistically supported by (1) reasonable waveform misfit ($R = 0.05$ in Figure 3b), (2) proper constraints on the model with good azimuth coverage [Kim et al., 2012], and (3) appropriate 3-D Green’s functions taking into account accurate source location and propagation effects.

6. Discussion and Conclusion

In this study we assumed the monopole source model for explosive eruptions. There are other possible elementary acoustic sources such as a dipole and quadrupole [Woulff and McGetchin, 1976; Lighthill, 1978], which may play a role in producing infrasound during eruptions. The monopole source depends on the net volume changes associated with eruptions, whereas other source types (dipole and quadrupole) depend on the momentum, irrelevant to the net volume change. Hence, the monopole source model may be suited for explosive eruptions causing significant volume expansion. Our inversion method is, however, not limited to the monopole but is applicable to the dipole and quadrupole by incorporating them into equation (1) [Johnson et al., 2008; Kim et al., 2012]. In that case, the inversion method can be used to verify the appropriate acoustic source model for a range of volcanic eruption styles, provided adequate station coverage is available.

Our solution represents eruption volume flux history by removing path effects from the infrasound signal. Thus, application of the 3-D method will provide a substantial improvement in the accuracy of eruption source estimation (e.g., initial overpressure of a gas slug) [Vergniolle and Brandeis, 1996]. Volume flux can be converted into mass eruption rate (MER) with additional information of the mixture density [Ripepe et al., 2013]. MER is an invaluable input parameter for volcanic ash transport and dispersion models used to forecast the trajectory of ash clouds [Mastin et al., 2009], and thereby immediate and accurate determination of MER is critical for mitigating ash-cloud hazards to aviation and local communities. Laboratory jet experiments also suggested that MER-time history is as important as instantaneous MER in the development of jet flow structure [Chojnicki et al., 2015]. In comparison with acoustically derived MER using the 1-D method [Ripepe et al., 2013], our 3-D inversion will substantially improve the accuracy of time-varying MER estimation for unsteady explosive eruptions and can potentially be implemented in real time. Application of this approach to chemical and nuclear explosions will also provide valuable source information (e.g., the amount of energy released), particularly when the sound propagation is affected by obstructions (e.g., explosions in urban environments or mountainous regions).

Acknowledgments

The authors thank Sakurajima Volcano Observatory for their invaluable help including logistical support with the field experiment. We are grateful to Matthew Haney for constructive and helpful discussions. This work was made possible with financial support from National Science Foundation EAR grant 1331084 and the Geophysical Institute of the University of Alaska Fairbanks. The authors also thank Bernard Chouet and an anonymous reviewer for their helpful comments. The Editor thanks Bernard A. Chouet and Takao Ohminato for their assistance in evaluating this paper.

References


Erratum

In the originally published version of this article, an incorrect version of Figure 3 appeared. The figure has since been corrected, and this version may be considered the authoritative version of record.